Energy Consumption Scheduling of HVAC Considering Weather Forecast Error through Distributionally Robust Approach

Y. F. Du, L. Jiang, *Member, IEEE,* C. Duan, Y. Z. Li, and J. S. Smith

***Abstract*—In this paper, the distributionally robust optimization approach (DROA) is proposed to schedule the energy consumption of the heating, ventilation and air conditioning (HVAC) system with consideration of the weather forecast error. The maximum interval of the outdoor temperature is partitioned into subintervals, and the proposed DROA constructs the ambiguity set of the probability distribution of the outdoor temperature based on the probabilistic information of these subintervals of historical weather data. The actual energy consumption will be adjusted according to the forecast error and the scheduled consumption in real time. The energy consumption scheduling of HVAC through the proposed DROA is formulated as a nonlinear problem with distributionally robust chance constraints. These constraints are reformulated to be linear and then the problem is solved via linear programming. Compared with the method that takes into account the weather forecast error based on the mean and the variance of historical data, simulation results demonstrate that the proposed DROA effectively reduces the electricity cost with less computation time, and the electricity cost is reduced compared with the traditional robust method.**

***Index Terms*—Distributionally robust optimization, HVAC, energy consumption scheduling, demand response.**

NOMENCLATURE

U_t^i	The ith subinterval of the outdoor temperature at time slot t.
U_t^m	The maximum interval of the outdoor temperature at time slot t.
B_t^i	The ith sub-zone of the outdoor temperature and the effect of users' activities at time slot t.
B_t^m	The maximum zone of the outdoor temperature and the effect of users' activities at time slot t.
$\mathcal{P}_t^0$	The set of all the probability distributions of uncertain variables.
$\mathcal{P}_t^1, \mathcal{P}_t^2$	The ambiguity set of the probability distribution of the outdoor temperature.
$\mathcal{P}_t^3$	The ambiguity set of the probability distribution of the outdoor temperature and the effect of users' activities.
$\mathbb{P}_t$	The probability distribution of uncertain variables.
t	Index of time slot.
i	Index of subinterval of the maximum interval of the outdoor temperature.
m	The total number of temperature subintervals.
ξ_t	The actual outdoor temperature.
μ_t	The forecast outdoor temperature.
σ_t^2	The variance of the outdoor temperature.
l_t^i	The lower bound of U_t^i.
u_t^i	The upper bound of U_t^i.
p_t^i	The probability of $\xi_t \in U_t^i$.
C	Thermal capacity of HVAC.
R	Thermal resistance.
η	Coefficient of performance of HVAC.
θ_t	The indoor temperature at time slot t.
θ^{min}	The lower bound of the indoor temperature.
θ^{max}	The upper bound of the indoor temperature.
θ^{best}	Users' preferred indoor temperature.
κ	The penalty factor of the deviation of users' preferred temperature.
w_1	The importance factor of Cost (electricity cost).
w_2	The importance factor of Num_VioTem (number of violations of comfortable temperatures).
w_3	The importance factor of Time (computation time).
Δt	The time period in a time slot.
q_t	The real-time power consumption of HVAC.
q_t^{ref}	The reference power consumption of HVAC.
q^{max}	The upper bound of power consumption of HVAC.
e_t	The electricity price at time slot t.
T	The scheduling horizon.
ε	The violation probability of the bounds of power consumption of HVAC.
s_e	Index of scenario of the electricity price.
S_e	The total number of scenarios of the electricity price.
φ_t	The effect of users' activities on the indoor temperature.
ϕ_t	The forecast effect of users' activities on the indoor temperature.
φ_t^{max}	The maximum effect of users' activities on the indoor temperature.

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$\varphi_t^{\min}$	The minimum effect of users' activities on the indoor temperature.
β, h, y	
$\boldsymbol{y}, \lambda_i, \lambda$	Auxiliary variables.
τ_0, τ_1	

I. Introduction

DEMAND response (DR) aims to schedule the energy consumption of appliances in response to varying electricity prices over time, or to incentive payments, or when the system reliability is jeopardized [1] [2]. DR can help improve the efficiency and the reliability of the power system [3]. Among the main potential DR resources are heating, ventilation and air conditioning (HVAC) systems because of their relatively large energy consumption. The energy consumption of HVAC accounts for about 40% of the energy consumption in a building [4] and can be up to 60% [5]. Moreover, its energy consumption has direct impact on the indoor temperature and thus significantly affects the users' comfort. The energy consumption scheduling of HVAC in DR is to minimize the electricity cost according to electricity prices with the indoor temperature maintained in a comfortable zone [6].

The energy consumption of HVAC is usually scheduled based on the forecast outdoor temperature [6]. The forecast error is one of the main factors which affect the indoor temperature and may cause the violation of the comfortable temperature zone [7]. Many studies have been carried out to deal with the forecast error in the energy consumption scheduling of HVAC [8]–[10]. The forecast error of the outdoor temperature, i.e. the uncertainty of the outdoor temperature is considered through the stochastic optimization approach (SOA) with a certain probability distribution in [10] and through the robust optimization approach (ROA) with a temperature interval in [9]. The SOA requires the exact probability distribution of the uncertain variable [11]. However, since the probability distribution of the outdoor temperature can only be estimated, the distribution itself is uncertain. The deviation between the actual distribution and the adopted distribution by the SOA may result in suboptimal solutions [12]. Though the ROA does not require the probability distribution [13] [14], it may be overly conservative since only the maximum forecast error is considered [15]. The distributionally robust optimization approach (DROA) combines the advantages of both the SOA and the ROA [16]. It does not require the exact probability distribution of the uncertain variable and its conservativeness is reduced with the probabilistic information observed from historical data [16]. With the mean and the variance extracted from historical data, this approach has been currently applied in the reserve scheduling problem in the power system [16]. The uncertainty of renewable energy is considered and the reserve scheduling problem is solved based on semidefinite programming (SDP) [16].

In this paper, a newly proposed DROA based on the probabilistic information of subintervals of the outdoor temperature is adopted to schedule the energy consumption of HVAC. Different from the DROA based on the mean and the variance, more information is extracted from historical weather data. The proposed DROA partitions the maximum interval of the outdoor temperature into subintervals, and it constructs the ambiguity set of the probability distribution of the outdoor temperature taking into account the probabilistic information of historical data within these subintervals. To eliminate the effect of the deviation between the actual outdoor temperature and the forecast one, the actual energy consumption of HVAC is proposed to be adjusted in real time based on the scheduled consumption and the forecast error. With the consideration of the ambiguity set of the outdoor temperature, the energy consumption scheduling of HVAC is formulated as a nonlinear problem with distributionally robust chance constraints. These constraints are reformulated to be linear and the energy consumption scheduling of HVAC is obtained through linear programming (LP). The proposed DROA based on the probabilities of subintervals is compared with the DROA based on the mean and the variance and the ROA in the electricity cost, the users' comfort and the computation time.

The rest of this paper is organized as follows. The problem formulation of the energy consumption scheduling of HVAC based on the proposed DROA is presented in Section II. Section III proposes the solution theorem to reformulate the distributionally robust chance constraints to be linear so that the problem can be solved by LP. Simulation results are presented in Section IV which compare the proposed DROA with the other methods. Finally, conclusions are presented in Section V.

II. Problem Formulation

In this section, the energy consumption scheduling of HVAC based on the proposed DROA is formulated. Firstly, the weather forecast error is modeled, and the distributionally robust chance constraints of the energy consumption of HVAC are introduced. Then the complete optimization model of the energy consumption scheduling of HVAC with consideration of the forecast error is presented.

A. Model of weather forecast error

The weather forecast predicts an outdoor temperature but the actual temperature may be different from this forecast. The proposed DROA partitions the maximum interval of the outdoor temperature into nested subintervals as follows

$$\begin{array}{l} U_t^i = \{\xi_t \in \mathcal{R} \mid l_t^i \le \xi_t \le u_t^i\}, i = 1, \cdots, m \\ U_t^1 \subseteq \cdots \subseteq U_t^m \end{array} \tag{1}$$

where ξ_t denotes the outdoor temperature at time slot t, U_t^m denotes its maximum interval, and m denotes the number of temperature intervals. The lower and upper bounds of subintervals are represented as

$$\begin{array}{l} l_t^{m-i} = l_t^m + i \cdot \frac{u_t^m - l_t^m}{2m-1} \\ u_t^{m-i} = u_t^m - i \cdot \frac{u_t^m - l_t^m}{2m-1}, i = 1, 2, \cdots, m-1. \end{array} \tag{2}$$

The example of temperature intervals with $m = 3$ is shown in Fig. 1. Based on the historical weather data, the ambiguity set of the probability distribution of the outdoor temperature is constructed with the consideration of the maximum interval

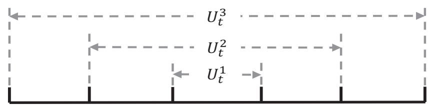

Fig. 1. Nested temperature intervals with $m = 3$

TABLE I
PARAMETERS OF HVAC SYSTEM

Parameter	Units	Description
C	kWh/°F	Thermal capacity of HVAC
R	°F/kW	Thermal resistance
η	none	Coefficient of performance of HVAC. This value is positive for cooling and negative for heating.

of the outdoor temperature and the probabilistic information of subintervals

$$\mathcal{P}_t^1 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(U_t^m) \;\middle|\; \begin{array}{l} \mathbb{E}_{\mathbb{P}_t}\{\xi_t\} = \mu_t \\ \mathbb{P}_t\{\xi_t \in U_t^i\} = p_t^i, \; i = 1, \cdots, m \\ U_t^1 \subseteq \cdots \subseteq U_t^m, p_t^m = 1 \end{array} \right\} \tag{3}$$

where μ_t denotes the forecast temperature, and p_t^i denotes the probability of $\xi_t \in U_t^i$ and its value is obtained from historical weather data. $\mathbb{P}_t$ denotes the probability distribution of ξ_t and $\mathcal{P}_t^0(U_t^m)$ denotes the set of all the probability distributions supported on U_t^m.

B. Distributionally robust chance constraints

To eliminate the effect of the weather forecast error, the energy consumption of HVAC is proposed to be adjusted in real time according to the deviation of the actual outdoor temperature from the forecast one, and the energy consumption is scheduled with the consideration of this adjustment through distributionally robust chance constraints. Based on the model of HVAC system, the indoor temperature is [6] [10]

$$\theta_t = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - \xi_{t-1} + \eta \cdot R \cdot q_{t-1}) \tag{4}$$

where θ_t denotes the indoor temperature at time slot t, q_{t-1} denotes the power of the energy consumption of HVAC at time slot $t-1$ and Δt denotes the time period. The parameters C, R and η of HVAC system are summarized in Table I [6]. The indoor temperature will remain the same when the energy consumption power of HVAC is adjusted in real time according to the forecast error of the outdoor temperature

$$q_t = q_t^{\text{ref}} + \frac{1}{\eta \cdot R} \cdot (\xi_t - \mu_t) \tag{5a}$$

$$\mathbb{P}_t\{q_t \geq 0\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{5b}$$

$$\mathbb{P}_t\{q_t \leq q^{\max}\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{5c}$$

where q_t^{ref} denotes the reference power of energy consumption, i.e. the energy consumption schedule, which is proposed in advance. How the reference energy consumption of HVAC is determined will be introduced in the next section. $q^{\max}$ denotes the maximum power of HVAC. The distributionally robust chance constraints (5b) and (5c) show that both probabilities of the power consumption satisfying the upper limit and the lower limit should be no smaller than $1 - \varepsilon$ for all the probability distributions of the outdoor temperature in the ambiguity set $\mathcal{P}_t^1$.

C. Complete optimization model

Based on the electricity price and the users' predefined comfort zone for the indoor temperature, the energy consumption of HVAC is scheduled aiming to minimize the electricity cost with users' satisfaction of the indoor temperature, which is formulated as

$$\min_{q_t^{\text{ref}}} \mathbb{E}\{\sum_{t=1}^{T}(e_t \cdot q_t \cdot \Delta t)\} \tag{6a}$$

$$q_t = q_t^{\text{ref}} + \frac{1}{\eta \cdot R} \cdot (\xi_t - \mu_t) \tag{6b}$$

$$\theta_t = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - \xi_{t-1} + \eta \cdot R \cdot q_{t-1}) \tag{6c}$$

$$\theta^{\min} \leq \theta_t \leq \theta^{\max} \tag{6d}$$

$$\mathbb{P}_t\{q_t \geq 0\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{6e}$$

$$\mathbb{P}_t\{q_t \leq q^{\max}\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{6f}$$

where e_t denotes the electricity price at time slot t and T denotes the scheduling horizon. $\theta^{\min}$ and $\theta^{\max}$ denote the lower bound and the upper bound of the comfortable temperature zone, respectively. The energy consumption is scheduled taking into account the weather forecast error based on the nested intervals of the outdoor temperature, and the effect of the weather forecast error is eliminated through the real-time adjustment of the energy consumption with the consideration of the distributionally robust chance constraints.

This energy consumption model can be applied to the HVAC system with only on-off control action, i.e. the power of HVAC can be either $q^{\max}$ or 0. Firstly, the power of HVAC's energy consumption is obtained based on the proposed system model and this power will last for Δt. Note that the effect of energy consumption on the indoor temperature is the same when the energy consumption of HVAC in Δt is the same. When the HVAC system with only on-off control action is adopted, the time when HVAC is on can be adjusted to satisfy the same energy consumption in Δt, i.e. the time when HVAC is on is $\frac{q_t \cdot \Delta t}{q^{\max}}$.

III. SOLUTION APPROACH

In this section, the distributionally robust chance constraints (6e) and (6f) are reformulated to be tractable and linear based on the theorem proposed below, and then the energy consumption scheduling of HVAC based on the proposed DROA can be solved through LP. For convenience, these two constraints are presented in a uniform form

$$\mathbb{P}_t\{a_t \cdot \xi_t \leq b_t\} \geq 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^1 \tag{7}$$

where a_t and b_t for (6e) and (6f) are specified in Table II. It has been proved in [17] that

$$\mathbb{P}_t\text{-CVaR}_\varepsilon(a_t \cdot \xi_t - b_t) \leq 0 \Rightarrow \mathbb{P}_t\{a_t \cdot \xi_t \leq b_t\} \geq 1 - \varepsilon$$
$$\mathbb{P}_t\text{-CVaR}_\varepsilon(a_t \cdot \xi_t - b_t)$$

TABLE II
EXPRESSIONS OF a_t AND b_t

Constraint	a_t	b_t
(6e)	$-\frac{1}{\eta \cdot R}$	$q_t^{\mathrm{ref}} - \frac{1}{\eta \cdot R} \cdot \mu_t$
(6f)	$\frac{1}{\eta \cdot R}$	$q_t^{\max} - q_t^{\mathrm{ref}} + \frac{1}{\eta \cdot R} \cdot \mu_t$

$$= \inf_{\beta \in \mathcal{R}} \left\{ \beta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}_t} \{(a_t \cdot \xi_t - b_t - \beta)^+\} \right\} \quad (8)$$

where $(x)^+ = \max(x, 0)$, and CVaR denotes conditional value at risk and it is introduced based on value at risk (VaR). VaR is the value satisfying the probability that $a_t \cdot \xi_t - b_t$ is above VaR is at most ε, and CVaR is defined as the mean of $a_t \cdot \xi_t - b_t$ on the tail distribution exceeding VaR [18]. More about CVaR can be found in [18]. According to (8), constraint (7) is satisfied if

$$\mathbb{P}_t\text{-CVaR}_\varepsilon(a_t \cdot \xi_t - b_t) \le 0, \ \forall \mathbb{P}_t \in \mathcal{P}_t^1. \quad (9)$$

Theorem 1: When the ambiguity set $\mathcal{P}_t^1$ is constructed, the distributionally robust constraint (9) is satisfied if and only if there exist y, β and $\lambda_i, i = 1, \cdots, m$, such that

$$\beta + \frac{1}{\varepsilon} \cdot (\mu_t \cdot y + \sum_{i=1}^m \lambda_i \cdot p_t^i) \le 0 \quad (10a)$$

$\forall i = 1, \cdots, m:$

$$y \cdot l_t^i + \sum_{j=i}^m \lambda_j \ge 0 \quad (10b)$$

$$y \cdot u_t^i + \sum_{j=i}^m \lambda_j \ge 0 \quad (10c)$$

$$y \cdot l_t^i + \sum_{j=i}^m \lambda_j - (a_t \cdot l_t^i - b_t - \beta) \ge 0 \quad (10d)$$

$$y \cdot u_t^i + \sum_{j=i}^m \lambda_j - (a_t \cdot u_t^i - b_t - \beta) \ge 0 \quad (10e)$$

Proof: First note that constraint (9) is equivalent to

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^1} \inf_{\beta \in \mathcal{R}} \left\{ \beta + \frac{1}{\varepsilon} \mathbb{E}_{\mathbb{P}_t} \{(a_t \cdot \xi_t - b_t - \beta)^+\} \right\}$$
$$= \inf_{\beta \in \mathcal{R}} \left\{ \beta + \frac{1}{\varepsilon} \sup_{\mathbb{P}_t \in \mathcal{P}_t^1} \mathbb{E}_{\mathbb{P}_t} \{(a_t \cdot \xi_t - b_t - \beta)^+\} \right\} \le 0 \quad (11)$$

where the interchangeability of sup and inf is guaranteed by a stochastic saddle point theorem [19]. To reformulate constraint (11), the following worst-case expectation needs to be evaluated

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^1} \mathbb{E}_{\mathbb{P}_t} \{(a_t \cdot \xi_t - b_t - \beta)^+\}. \quad (12)$$

Note that the probability distribution of the outdoor temperature is not known and there are infinitely many possible distributions which form an ambiguity set. The infinite dimensional linear optimization problem (12) is equivalent to

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^0(U_t^m)} \int_{U_t^m} (a_t \cdot \xi_t - b_t - \beta)^+ \mathbb{P}_t(d\xi_t) \quad (13a)$$

$$\text{s.t.} \int_{U_t^m} \xi_t \mathbb{P}_t(d\xi_t) = \mu_t \quad (13b)$$

$$\int_{U_t^m} I_{U_t^i} \mathbb{P}_t(d\xi_t) = p_t^i, \ i = 1, \cdots, m \quad (13c)$$

where $I_{U_t^i} = 1$ when $\xi_t \in U_t^i$, otherwise $I_{U_t^i} = 0$. By introducing dual variables y and λ_i, (13) is reformulated as

$$\inf_{y, \boldsymbol{\lambda}} \ \mu_t \cdot y + \sum_{i=1}^m \lambda_i \cdot p_t^i \quad (14a)$$

$$\text{s.t. } y \in \mathcal{R}, \lambda_i \in \mathcal{R}, \ i = 1, \cdots, m \quad (14b)$$

$$\inf_{\xi_t \in U_t^m} \left\{ y \cdot \xi_t + \sum_{i=1}^m \lambda_i \cdot I_{U_t^i} - (a_t \cdot \xi_t - b_t - \beta)^+ \right\} \ge 0. \quad (14c)$$

Now the problem has been reformulated to a finite dimensional optimization problem. Since U_t^m can be partitioned into m mutually disjoint sets $\mathcal{R}_t^1 = U_t^1$, $\mathcal{R}_t^i = U_t^i \setminus U_t^{i-1}$, $i = 2, \cdots, m$, (14c) is equivalent to

$$\inf_{\xi_t \in \mathcal{R}_t^i} \left\{ y \cdot \xi_t + \sum_{j=i}^m \lambda_j - (a_t \cdot \xi_t - b_t - \beta)^+ \right\} \ge 0, \quad \forall i = 1, \cdots, m \quad (15)$$

which can be further equivalently converted to

$$\inf_{\xi_t \in U_t^i} \left\{ y \cdot \xi_t + \sum_{j=i}^m \lambda_j - (a_t \cdot \xi_t - b_t - \beta)^+ \right\} \ge 0, \quad \forall i = 1, \cdots, m \quad (16)$$

considering that $U_t^i \supseteq \mathcal{R}_t^i$, and that the infimum of (16) is attained on the boundary of U_t^i and $\mathcal{R}_t^i$ contains the boundary of U_t^i. Constraint (16) is satisfied if and only if $\forall i = 1, \cdots, m$,

$$y \cdot l_t^i + \sum_{j=i}^m \lambda_j \ge 0 \quad (17a)$$

$$y \cdot u_t^i + \sum_{j=i}^m \lambda_j \ge 0 \quad (17b)$$

$$y \cdot l_t^i + \sum_{j=i}^m \lambda_j - (a_t \cdot l_t^i - b_t - \beta) \ge 0 \quad (17c)$$

$$y \cdot u_t^i + \sum_{j=i}^m \lambda_j - (a_t \cdot u_t^i - b_t - \beta) \ge 0 \quad (17d)$$

By substituting (14) into (11) together with (17), the proposed theorem is proved. ■

The CVaR approximation is the first step of our method and our main contribution focuses on the reformulation of the constraint after the CVaR approximation. In this reformulation, it is noted that the probability distribution of the outdoor temperature is unknown and there are infinitely many possible distributions which form an ambiguity set, and we propose a method which immunizes the solution of the problem against all possible distributions after the CVaR approximation. Since the constraint (9) is more conservative than the distributionally chance constraint (7), the electricity cost will be higher after

TABLE III
TIME OF USE ELECTRICITY PRICES

Time	12am-2am	2am-6am	6am-10am	10am-12pm
Price($/kWh)	0.00493	0.00493	0.05040	0.05040
Time	12pm-2pm	2pm-8pm	8pm-10pm	10pm-12am
Price($/kWh)	0.05040	0.09761	0.05040	0.00493

the CVaR approximation. Although the CVaR constraint brings conservatism when approximating (7), it is superior to the original constraint from other aspects. Firstly, as shown in ***Theorem 1***, the distributionally robust CVaR constraint admits tractable convex reformulation. Secondly, the CVaR constraint imposes higher penalties on larger constraint violations [20]. Therefore, the CVaR constraint confines both the probability and the severity of constraint violations.

IV. SIMULATION RESULTS

This section presents the simulation results to verify the effectiveness of the proposed DROA in the scheduling of H-VAC's energy consumption. The parameters of HVAC system C, R and η are assumed to be 0.33 kWh/°F, 13.5 °F/kW and 2.2, respectively [6] [9]. The maximum power of HVAC is assumed to be 1.75 kW [6] and the scheduling period Δt is 30 minutes [10]. The energy consumption is scheduled 12 hours ahead with $T = 24$. The electricity price based on the time of use, as shown in Table III, is adopted from the Austin Energy Company [21]. The outdoor temperature in Austin from 12 pm August 6th 2013 to 12pm August 9th 2013 is assumed to be the forecast outdoor temperature [22]. Based on the normal distribution with the forecast temperature as the mean and 2.5 as the standard deviation [9], 10000 samples are taken to simulate the historical data and to construct the ambiguity set $\mathcal{P}_t^1$. It is noted that the DROA does not require the probability distribution of the outdoor temperature and the normal distribution is used to generate historical data. In practice, the forecast and the actual temperature values are both recorded as the historical data and $\mathcal{P}_t^1$ is constructed based on these historical weather data. All the simulations are implemented in MATLAB with YALMIP [23] as the modelling tool and SeDuMi [24] as the solver running on an Intel Core-i3 3.3-GHz personal computer with 8 GB RAM. In practice, the energy consumption scheduling of HVAC is solved by a controller in DR and this controller is with enough computation power.

For convenience, the proposed DROA together with the other two methods are listed and referred to as M1-M3 as shown below. Firstly, the proposed DROA is compared with other two methods in the electricity cost, the users' comfort and the computation time. Then the impacts of m, ε and the comfortable temperature zone on the performance of the proposed DROA are investigated, respectively. Furthermore, the proposed DROA is extended to take into account more uncertainties.

- M1: Based on the proposed DROA considering the probabilistic information of subintervals of the outdoor temperature, the problem of energy consumption scheduling is reformulated to be LP through the proposed ***Theorem 1***.
- M2: The energy consumption is scheduled based on the DROA considering the mean and the variance of historical data, which is formulated as the same as (6) except that the ambiguity set $\mathcal{P}_t^1$ is replaced by

$$\mathcal{P}_t^2 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(U_t^m) \;\middle|\; \begin{array}{l} \mathbb{E}_{\mathbb{P}_t}\{\xi_t\} = \mu_t \\ \mathbb{P}_t\{\xi_t \in U_t^m\} = 1, \\ \mathbb{E}_{\mathbb{P}_t}\{(\xi_t - \mu_t)^2\} = \sigma_t^2 \end{array} \right\} \tag{18}$$

where σ_t^2 denotes the variance of the outdoor temperature obtained from historical weather data. Then the problem of energy consumption scheduling is reformulated to be SDP based on the theorem below.

Theorem 2 [25]: When the ambiguity set $\mathcal{P}_t^2$ is constructed, the distributionally robust constraint (9) is satisfied if and only if there exist $y, \beta, h, \lambda, \tau_0$ and τ_1, such that

$$\beta + \frac{1}{\varepsilon} \cdot (h + \mu_t \cdot y + \lambda \cdot \sigma_t^2 + \lambda \cdot \mu_t^2) \leq 0 \tag{19a}$$

$$\tau_0 \geq 0, \tau_1 \geq 0 \tag{19b}$$

$$\mathbf{M} + \tau_0 \cdot \mathbf{W} \succeq 0 \tag{19c}$$

$$\mathbf{M} + \tau_1 \cdot \mathbf{W} - \mathbf{H} \succeq 0 \tag{19d}$$

$$\mathbf{M} = \begin{bmatrix} \lambda & \frac{y}{2} \\ \frac{y}{2} & h \end{bmatrix}, \mathbf{W} = \begin{bmatrix} 1 & -\frac{l_t^m + u_t^m}{2} \\ -\frac{l_t^m + u_t^m}{2} & l_t^m \cdot u_t^m \end{bmatrix} \tag{19e}$$

$$\mathbf{H} = \begin{bmatrix} 0 & \frac{a_t}{2} \\ \frac{a_t}{2} & -b_t - \beta \end{bmatrix} \tag{19f}$$

The proof of ***Theorem 2*** is shown in Appendix. Note that M2 based on the mean and the variance of the outdoor temperature reformulates the problem to be SDP, which is more computationally expensive than LP based on the proposed M1.

- M3: The energy consumption is scheduled based on the ROA, which is formulated as the same as (6) except that constraints (6e) and (6f) are replaced by conventional robust constraints

$$q_t \geq 0, \forall \xi_t \in U_t^m \tag{20a}$$

$$q_t \leq q^{\max}, \forall \xi_t \in U_t^m. \tag{20b}$$

Note that the reference energy consumption is proposed taking into account the weather forecast error and that the actual energy consumption will be adjusted in real time based on the reference energy consumption and the weather forecast error as shown in (5a). The actual energy consumption is set to be $q^{\max}$ and 0 under the circumstances where the adjusted energy consumption exceeds these two values, respectively.

A. Comparison between the proposed DROA and the other methods

In this section, the proposed DROA is compared with the other two methods with $m = 15, \varepsilon = 0.005$, [60° F, 70° F] as the comfortable temperature zone and 70 °F as the start indoor temperature. Firstly, the simulation results in a scheduling cycle from 12 pm to 12 am on August 6th 2013

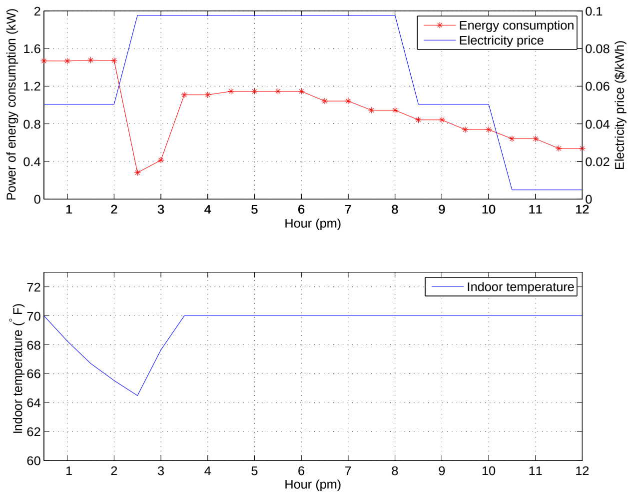

Fig. 2. Energy consumption schedule and indoor temperature based on M1

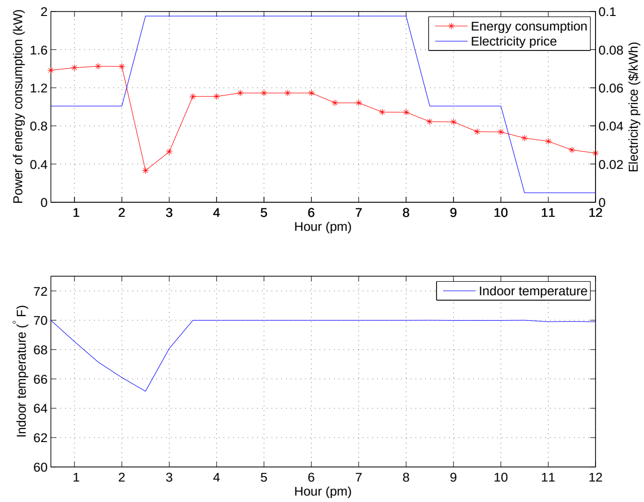

Fig. 3. Energy consumption schedule and indoor temperature based on M2

are demonstrated, then the simulation results in consecutive cycles from 12 pm August 6th 2013 to 12pm August 9th 2013 are presented.

1) In a scheduling cycle: The energy consumption schedule and the performances of this energy consumption schedule under one test sample of the outdoor temperature and 10000 test samples are presented.

- Energy consumption schedule:
  Fig. 2-4 show the energy consumption schedule and the indoor temperature based on M1, M2 and M3, respectively. From 1pm to 3pm, it can be seen from Fig. 2-4 that the energy consumption of HVAC is largely scheduled in periods with low electricity price to pre-cool the indoor temperature. Then the energy consumption can be saved in periods with high electricity price. Fig. 5 summarizes the energy consumption schedules based on the different methods and the dash line presents the maximum power consumption of HVAC $q^{\max}$. Fig. 5 shows that M3 considers the maximum error of the weather forecast and its maximum and minimum energy consumptions are far from $q^{\max}$ and 0, respectively. It can be seen from Fig. 5 that M3 is the most conservative and M1 is less conservative than M2.
- Test with a sample of the outdoor temperature:
  Based on the normal distribution with the forecast temperature as the mean and 2.5 as the standard deviation, a sample of the outdoor temperature, as shown in Fig. 6, is taken to test the energy consumption schedules in Fig. 5. Fig. 7 and Fig. 8 show the adjusted energy consumption and the indoor temperature under this test outdoor temperature based on the three methods, respectively. The adjusted energy consumption is obtained based on (5a). For all the methods, the adjusted energy consumption is within the limits and it is the same as the actual energy consumption. The electricity costs of the actual energy consumption are \$0.809, \$0.811 and \$0.815 for M1-M3, respectively, and the electricity cost of M1 is reduced compered with M2 and M3. Fig. 8 shows that the actual indoor temperature is following the reference indoor temperature based on M1, M2 and M3 and the indoor temperature is within the comfortable temperature zone. The reference indoor temperature is the indoor temperature that is obtained with the reference energy consumption 12 hours ahead. Based on M1, M2 and M3, the weather forecast error is considered in the scheduling process and its effect on the indoor temperature is eliminated through the real-time adjustment of energy consumption, and the electricity cost of the proposed M1 is smallest.

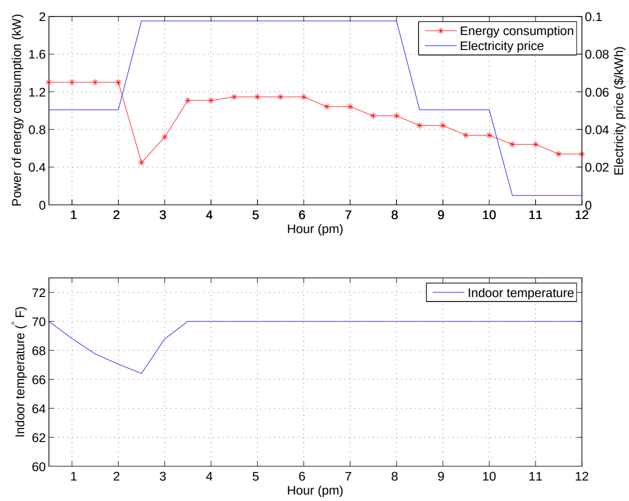

Fig. 4. Energy consumption schedule and indoor temperature based on M3

- Test with 10000 samples of the outdoor temperature:
  Based on the normal distribution with the forecast temperature as the mean and 2.5 as the standard deviation, 10000 samples of the outdoor temperature are taken to

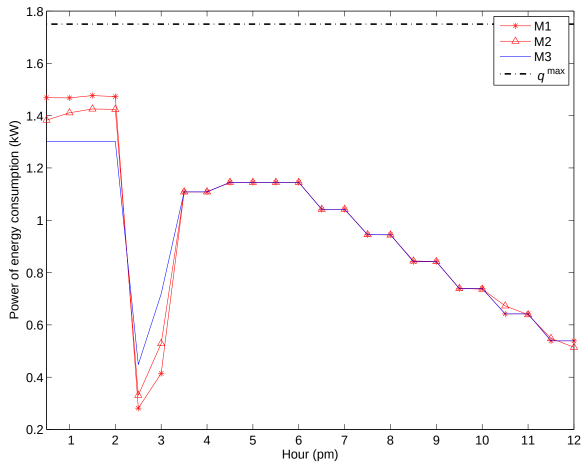

Fig. 5. Energy consumption schedules based on different methods

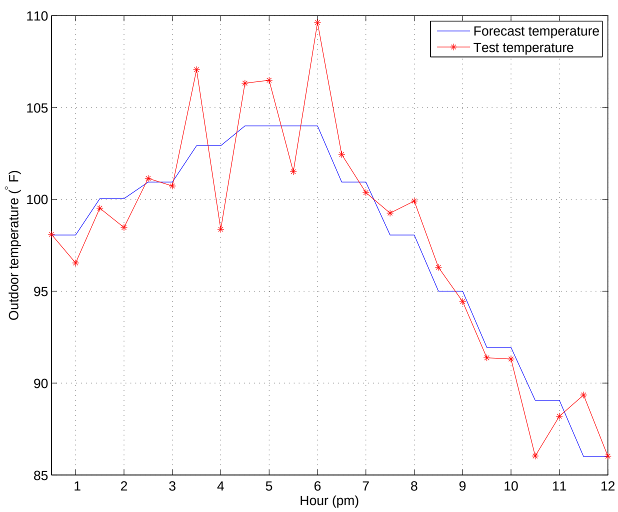

Fig. 6. The outdoor temperature

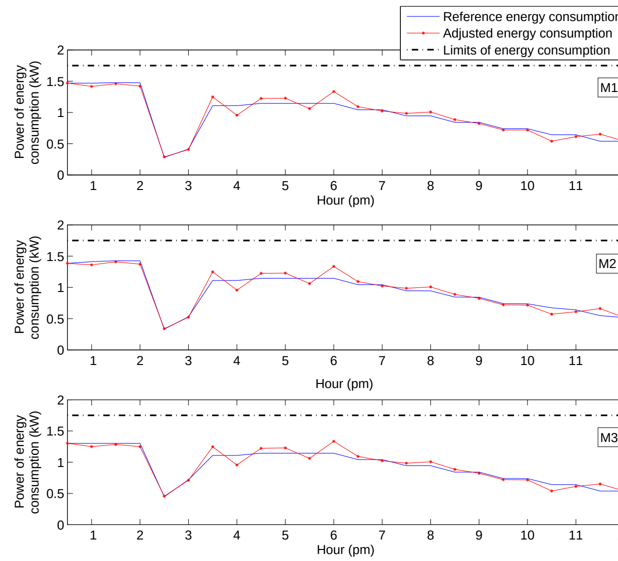

Fig. 7. The adjusted energy consumption

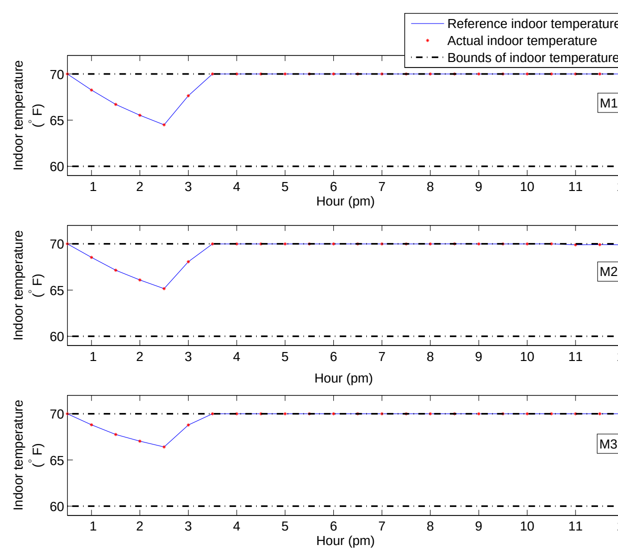

Fig. 8. The indoor temperature

test the three methods. Note that 10000 samples are used as historical data to construct the ambiguity set and the new 10000 samples are taken for testing. It is remarkable that the DROA does not require the prior knowledge about the distribution of the outdoor temperature, and the probability information in the ambiguity set is extracted from historical data. Any probability distribution can be used to test the DROA only if the data employed in the ambiguity set construction and the performance evaluation is sampled from the same distribution. The average electricity cost of the actual energy consumption (Cost), the number and the maximum of violations from the comfortable temperature zone (Num_VioTem and VioTem), and the computation time (Time) are summarized in Table IV for the three methods. It can be seen from Table IV that the indoor temperature is always within the comfortable zone for M3 while M3 pays the highest electricity cost. In comparison with M2, it is noted that the proposed M1 takes less time to obtain the optimal energy consumption schedule of HVAC and the electricity cost of M1 is also less. The computation time is 0.5110 seconds for M1 and it is 5.7651 seconds for M2. With the probabilistic information of subintervals taken into account and the reformulation of the problem as LP, the proposed M1 helps reduce the electricity cost with less computation time compared with M2 based on the mean and the variance of historical weather data and the reformulation of the problem as SDP.

Table V shows numbers of situations (Num_VioEneL and Num_VioEneH) where the adjusted energy consumption violates the lower and upper limits of the energy consumption of HVAC within $T = 24$ un-

TABLE IV
PERFORMANCES OF THE THREE METHODS IN A SCHEDULING CYCLE

Method	Cost ($)	Num_VioTem	VioTem (°F)	Time (s)
M1	0.793	22	0.3579	0.5110
M2	0.796	3	0.0593	5.7651
M3	0.799	0	0	0.1494

TABLE V
VIOLATIONS OF ENERGY CONSUMPTION LIMITS

Method	M1	M2
Num_VioEneL	5	1
Prob_VioEneL	0.0021%	0.0004%
Num_VioEneH	22	5
Prob_VioEneH	0.0092%	0.0021%

der 10000 tests for M1 and M2. The probabilities of deviations of the lower and upper limits per time slot and per test (Prob_VioEneL and Prob_VioEneH) can be estimated as Num_VioEneL/(24×10000) and Num_VioEneH/(24×10000), respectively. The estimated probabilities are all less than $\varepsilon = 0.005$, which shows that the distributionally robust chance constraints (6e) and (6f) are satisfied based on M1 and M2.

2) In consecutive scheduling cycles: The performance of the proposed DROA is tested under consecutive cycles from 12 pm August 6th 2013 to 12pm August 9th 2013 with 70 °F as the start indoor temperature, and the indoor end temperature of the previous cycle will be the indoor start temperature of the next cycle. The simulation results are shown in Table VI. It can be seen from Table VI that the electricity cost is reduced through the proposed M1 and the computation time of M1 is largely decreased compared with M2.

B. The proposed DROA with different parameters

In this section, the impacts of m, ε and the comfortable temperature zone on the performance of the proposed DROA are investigated, respectively, in the scheduling cycle form 12 pm to 12 am on August 6th 2013.

1) With different m: M1 with different m is tested with 10000 samples of the outdoor temperature under the condition of $\varepsilon = 0.005$ and [60° F, 70° F] as the comfortable temperature zone. The performances of M1 with different m are summarized in Table VII. It can be seen from Table VII that the indoor temperature is mostly within the comfortable zone. With the increase of m, i.e. with more probabilistic information of the outdoor temperature taken into account, the electricity cost is decreasing and the computation time is increasing.

2) With different ε: M1 with different ε is tested with 10000 samples of the outdoor temperature under the condition of $m = 15$ and [60° F, 70° F] as the comfortable temperature zone. The performances of M1 with different ε are summarized in Table VIII. It can be seen from Table VIII that with the increase of ε, the electricity cost is decreasing and the probability of the violation of the comfortable temperature zone is increasing.

3) With different comfortable temperature zone: M1 with different comfortable temperature zones is tested with 10000 samples of the outdoor temperature under the condition of $m = 15$ and $\varepsilon = 0.005$. The performances of M1 with different comfortable temperature zones are summarized in Table IX. It can be seen from Table IX that the electricity cost increases when the comfortable temperature zone is narrowed.

With the consideration of the cost, the number of violations of the comfortable temperature zone and the computation time, genetic algorithm (GA) can be used to find the optimal combination of m, ε and the comfortable temperature zone. GA mimics the process of natural selection. After evaluating the fitness of each individual in generation, selecting individuals with high fitness, crossover and mutation, the new generation with better fitness is obtained, and the above process cycles until the individual with the satisfactory fitness is found [26]. To find the optimal combination of parameters, a random combination of m, ε and the comfortable temperature zone indicates an individual of GA, and the sum of the cost, the number of violations and the computation time with their corresponding importance factors indicates the fitness of GA, i.e. $w_1 \cdot \text{Cost} + w_2 \cdot \text{Num\_VioTem} + w_3 \cdot \text{Time}$ is the objective function of GA, where w_1, w_2 and w_3 are the importance factors. With m between 5 and 25, ε between 0.005 and 0.080, the lower limit of the comfortable temperature zone between 60°F and 68°F, and the upper limit of the comfortable temperature zone with the fixed 70°F, $m = 8, \varepsilon = 0.013$ and 67.5°F as the lower limit of the temperature zone is the optimal combination of parameters obtained through GA for the situation where the cost, the number of violations divided by 100 and the computation time are with the same importance. The number of violations is divided by 100 due to the magnitude difference among the number of violations, the cost and the computation time. The Cost, Num_VioTem, VioTem, Prob_VioEneL, Prob_VioEneH and Time are \$0.799, 2, 0.0438°F, 0, 0.0008% and 0.1336s, respectively, for the optimal combination of parameters.

C. The extension of the proposed DROA

In this section, the proposed DROA is extended to take into account uncertainties of the electricity price and the effect of users' activities on the indoor temperature and the deviation of users' preferred temperature. All the simulations are conducted in the scheduling cycle form 12 pm to 12 am on August 6th 2013.

1) Considering the uncertainty of the electricity price: The uncertainty of the electricity price is taken into account through Monte Carlo method with its probability distribution known, which is formulated as the same as (6) except that the objective function is changed to

$$\min_{q_t^{\text{ref}}} \mathbb{E}\{\frac{1}{S_e}\sum_{s_e=1}^{S_e}\sum_{t=1}^{T}(e_t \cdot q_t \cdot \Delta t)\}. \tag{21}$$

The proposed DROA with $m = 15, \varepsilon = 0.005$ and [60° F, 70° F] as the comfortable temperature zone is tested under 10000 samples of the outdoor temperature and the electricity price. The uncertainty of the electricity price is considered based on the normal distribution with the TOUP as the mean and 0.0003 as the standard deviation. The Cost, Num_VioTem,

TABLE VI
PERFORMANCES OF THE THREE METHODS IN CONSECUTIVE SCHEDULING CYCLES

Cycle	Method	Cost ($)	Num_VioTem	VioTem (°F)	Prob_VioEneL	Prob_VioEneH	Time (s)
1	M1	0.793	22	0.3579	0.0021%	0.0092%	0.5110
	M2	0.796	3	0.0593	0.0004%	0.0021%	5.7651
	M3	0.799	0	0	0	0	0.1494
2	M1	0.139	21	0.1713	0.0196%	0.0067%	0.4586
	M2	0.152	0	0	0	0	8.7290
	M3	0.159	0	0	0	0	0.1433
3	M1	0.802	14	0.1741	0.0013%	0.0058%	0.3636
	M2	0.804	1	0.0093	0.0008%	0.0004%	4.4855
	M3	0.808	0	0	0	0	0.2590
4	M1	0.135	7	0.0943	0.0229%	0.0025%	0.4945
	M2	0.146	0	0	0.0004%	0	7.8860
	M3	0.156	0	0	0	0	0.0977
5	M1	0.806	20	0.2444	0.0017%	0.0083%	0.3278
	M2	0.812	0	0	0	0	9.0289
	M3	0.812	0	0	0	0	0.1411
6	M1	0.121	27	0.1907	0.0267%	0.0038%	0.4658
	M2	0.128	0	0	0.0025%	0	5.9540
	M3	0.149	0	0	0	0	0.1499

TABLE VII
PERFORMANCES OF M1 WITH DIFFERENT m

m	Cost ($)	Num_VioTem	VioTem (°F)	Prob_VioEneL	Prob_VioEneH	Time (s)
5	0.794	7	0.1208	0.0008%	0.0029%	0.2823
10	0.794	6	0.2635	0.0004%	0.0042%	0.3141
15	0.793	22	0.3579	0.0021%	0.0092%	0.5110
20	0.793	21	0.1892	0.0025%	0.0088%	0.5658
25	0.793	33	0.2328	0.0029%	0.0138%	0.5897

TABLE VIII
PERFORMANCES OF M1 WITH DIFFERENT ε

ε	Cost ($)	Num_VioTem	VioTem (°F)	Prob_VioEneL	Prob_VioEneH	Time (s)
0.005	0.793	22	0.3579	0.0021%	0.0092%	0.5110
0.020	0.792	97	0.3535	0.0079%	0.0404%	0.4095
0.040	0.792	147	0.3210	0.0138%	0.0621%	0.2490
0.060	0.791	290	0.3957	0.0292%	0.1229%	0.2109
0.080	0.791	363	0.3692	0.0383%	0.1563%	0.2111

TABLE IX
PERFORMANCES OF M1 WITH DIFFERENT COMFORTABLE TEMPERATURE ZONES

Temperature Zone (°F)	Cost ($)	Num_VioTem	VioTem (°F)	Prob_VioEneL	Prob_VioEneH	Time (s)
[60-70]	0.793	22	0.3579	0.0021%	0.0092%	0.5110
[62-70]	0.793	14	0.2327	0.0021%	0.0058%	0.4985
[65-70]	0.794	9	0.1765	0.0017%	0.0037%	0.5576
[68-70]	0.801	6	0.0827	0	0.0025%	0.3451

VioTem, Prob_VioEneL, Prob_VioEneH and Time are \$0.793, 21, 0.2501°F, 0.0013%, 0.0088% and 0.4064s, respectively.

2) Considering the uncertainty of users' activities: The effect of users' activities on the indoor temperature as well as the uncertainty of the outdoor temperature is taken into account in the energy consumption scheduling of HVAC, which is formulated as

$$\min_{q_t^{\text{ref}}} \mathbb{E}\{\sum_{t=1}^{T}(e_t \cdot q_t \cdot \Delta t)\} \tag{22a}$$

$$q_t = q_t^{\text{ref}} + \frac{1}{\eta \cdot R} \cdot (\xi_t - \mu_t) + \frac{C}{\eta \cdot \Delta t} \cdot (\varphi_t - \phi_t) \tag{22b}$$

$$\theta_t = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - \xi_{t-1} + \eta \cdot R \cdot q_{t-1}) + \varphi_{t-1} \tag{22c}$$

$$\theta^{\min} \le \theta_t \le \theta^{\max} \tag{22d}$$

$$\mathbb{P}_t\{q_t \ge 0\} \ge 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^3 \tag{22e}$$

$$\mathbb{P}_t\{q_t \le q^{\max}\} \ge 1 - \varepsilon, \forall \mathbb{P}_t \in \mathcal{P}_t^3 \tag{22f}$$

$$\mathcal{P}_t^3 = \left\{ \mathbb{P}_t \in \mathcal{P}_t^0(B_t^m) \left| \begin{array}{l} \mathbb{E}_{\mathbb{P}_t}\{\boldsymbol{\omega}_t\} = \boldsymbol{\rho}_t \\ \boldsymbol{\omega}_t = (\xi_t, \varphi_t)^{\mathrm{T}}, \boldsymbol{\rho}_t = (\mu_t, \phi_t)^{\mathrm{T}} \\ \mathbb{P}_t\{\boldsymbol{\omega}_t \in B_t^i\} = p_t^i, \ i = 1, \cdots, m \\ B_t^i = \{\boldsymbol{\omega}_t \in \mathcal{R}^2 | \\ \qquad l_t^i \le \xi_t \le u_t^i \\ \qquad \varphi_t^{\min} \le \varphi_t \le \varphi_t^{\max}\} \\ B_t^1 \subseteq \cdots \subseteq B_t^m, p_t^m = 1 \end{array} \right. \right\} \tag{22g}$$

where φ_{t-1} is the effect of users' activities during $t-1$ time slot on the indoor temperature, and ϕ_t is the forecast effect of users' activities. (22b) is obtained based on $\theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - \mu_{t-1} + \eta \cdot R \cdot q_{t-1}^{\text{ref}}) + \phi_{t-1} = \theta_{t-1} - \frac{\Delta t}{C \cdot R} \cdot (\theta_{t-1} - \xi_{t-1} + \eta \cdot R \cdot q_{t-1}) + \varphi_{t-1}$. Therefore, the forecast errors of both the outdoor temperature and the effect of users' activities can be eliminated. Taking into account two uncertainties, the uniform form of distributionally robust chance constraints is changed to

$$\mathbb{P}_t\text{-CVaR}_\varepsilon(\boldsymbol{a}_t^{\mathrm{T}}\boldsymbol{\omega}_t \leq c_t) \leq 0,\ \forall \mathbb{P}_t \in \mathcal{P}_t^3 \tag{23}$$

where $\boldsymbol{a}_t$ is a column vector with two elements. (23) can be reformulated to be linear based on the theorem below and the proof is similar to the proof of ***Theorem 1***.

Theorem 3: When the ambiguity set $\mathcal{P}_t^3$ is constructed, the distributionally robust constraint (23) is satisfied if and only if there exist $\boldsymbol{y} \in \mathcal{R}^2$, $\beta \in \mathcal{R}$, and $\lambda_i \in \mathcal{R}, i = 1, \cdots, m$, such that

$$\beta + \frac{1}{\varepsilon} \cdot (\boldsymbol{\rho}_t^{\mathrm{T}}\boldsymbol{y} + \sum_{i=1}^{m} \lambda_i \cdot p_t^i) \leq 0 \tag{24a}$$

$\forall i = 1, \cdots, m:$

$$(l_t^i, \varphi_t^{\min})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j \geq 0 \tag{24b}$$

$$(l_t^i, \varphi_t^{\max})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j \geq 0 \tag{24c}$$

$$(u_t^i, \varphi_t^{\min})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j \geq 0 \tag{24d}$$

$$(u_t^i, \varphi_t^{\max})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j \geq 0 \tag{24e}$$

$$(l_t^i, \varphi_t^{\min})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j - ((l_t^i, \varphi_t^{\min})\boldsymbol{a}_t - c_t - \beta) \geq 0 \tag{24f}$$

$$(l_t^i, \varphi_t^{\max})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j - ((l_t^i, \varphi_t^{\max})\boldsymbol{a}_t - c_t - \beta) \geq 0 \tag{24g}$$

$$(u_t^i, \varphi_t^{\min})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j - ((u_t^i, \varphi_t^{\min})\boldsymbol{a}_t - c_t - \beta) \geq 0 \tag{24h}$$

$$(u_t^i, \varphi_t^{\max})\boldsymbol{y} + \sum_{j=i}^{m} \lambda_j - ((u_t^i, \varphi_t^{\max})\boldsymbol{a}_t - c_t - \beta) \geq 0 \tag{24i}$$

The proposed DROA with $m = 15, \varepsilon = 0.005$ and [60° F, 70° F] as the comfortable temperature zone is tested under 10000 samples of the outdoor temperature and the effect of users' activities. The effect of users' activities is sampled based on the normal distribution and the uniform distribution between $\varphi^{\min}$ and $\varphi^{\max}$, respectively, i.e. the proposed DROA is tested under two distributions of users' activities. Any probability distribution can be used to test the DROA only if the data employed in the ambiguity set construction and the performance evaluation is sampled from the same distribution, and the normal distribution and the uniform distribution are illustrated to test the performance of the proposed DROA. $\varphi^{\min}$ and $\varphi^{\max}$ are assumed to be [0.5, 0.5, 0.4, 0.4, 0, 0, 0, 0, 0.3, 0.3, 0.6, 0.6, 0.8, 0.8, 0.8, 0.8, 0, 0, 0, 0, 0, 0, 0, 0] and [1.3, 1.3, 0.8, 0.8, 0.4, 0.4, 0.2, 0.2, 0.5, 0.5, 1, 1, 1.2, 1.2, 1, 1, 0.6, 0.6, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2] in the scheduling horizon for both distributions. The Cost, Num_VioTem, VioTem, Prob_VioEneL, Prob_VioEneH and Time are \$0.931, 1, 0.0057°F, 0, 0.0004% and 0.9070s, respectively, for the normal distribution, and they are \$0.931, 5, 0.1215°F, 0, 0.0021% and 0.8426s, respectively, for the uniform distribution.

3) Considering the deviation of users' preferred temperature: Users may not only require the indoor temperature within the comfortable temperature zone but also have their preferred indoor temperature. The deviation of the indoor temperature from the preferred temperature is taken into account in the energy consumption scheduling of HVAC, which is formulated as the same as (6) except that the objective function is changed to

$$\min_{q_t^{\text{ref}}} \mathbb{E}\{\sum_{t=1}^{T} e_t \cdot q_t \cdot \Delta t + \kappa \cdot (\theta_t - \theta^{\text{best}})^2\} \tag{25}$$

where κ penalizes the deviation of the preferred indoor temperature and θ^{best} denotes users' preferred temperature.

The proposed DROA with $m = 15, \varepsilon = 0.005$, [60° F, 70° F] as the comfortable temperature zone and $\theta^{\text{best}} = 65°$ F is tested under 10000 samples of the outdoor temperature. The Cost, Num_VioTem, VioTem, Prob_VioEneL, Prob_VioEneH and Time are \$0.924, 0, 0, 0.0029%, 0.0121% and 0.7458s, respectively.

V. Conclusion

In this paper, the DROA based on the probabilistic information of subintervals of the outdoor temperature is proposed to schedule the energy consumption of HVAC. The simulation results have demonstrated that the proposed DROA helps reduce the electricity cost with less computation time compared with the DROA based on the mean and the variance of the outdoor temperature. The electricity cost is reduced as well compared with the traditional robust method. By increasing the number of temperature subintervals, i.e. by taking into account more information about the weather forecast error, the electricity cost of the proposed DROA is decreasing. The proposed DROA has proved effective in the energy consumption scheduling of HVAC with consideration of the weather forecast error.

Appendix
Proof of *Theorem 2*

Proof: The first two transformations are the same as (11) and (12) except that $\mathcal{P}_t^1$ is replaced by $\mathcal{P}_t^2$. (12) with $\mathbb{P}_t \in \mathcal{P}_t^2$ is equivalent to

$$\sup_{\mathbb{P}_t \in \mathcal{P}_t^0(U_t^m)} \int_{U_t^m} (a_t \cdot \xi_t - b_t - \beta)^+ \mathbb{P}_t(d\xi_t) \tag{26a}$$

$$\text{s.t.} \int_{U_t^m} \xi_t \mathbb{P}_t(d\xi_t) = \mu_t \tag{26b}$$

$$\int_{U_t^m} \mathbb{P}_t(d\xi_t) = 1 \tag{26c}$$

$$\int_{U_t^m} (\xi_t - \mu_t)^2 \mathbb{P}_t(d\xi_t) = \sigma_t^2 \tag{26d}$$

Through ***Theorem 3.7*** in [25] with dual variables y, h and λ introduced, ***Theorem 2*** is proved. ∎

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